Optimizing Spectrum Utilization in Dynamic RWA

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***Abstract*—While lightpath rearrangement has already been investigated by several authors for the dynamic RWA problem, we propose to revisit it with the goal of evaluating the minimum number of lightpath rearrangement it requires in order to remain with an optimized RWA provisioning, using ε-optimal solutions. Lightpath rearrangement is now made feasible with the use of colorless, directionless, and contentionless (CDC) reconfigurable optical add/drop multiplexers (ROADMs) in optical networks. While exact solution of the RWA problem was out of reach few years ago, it is now possible for fairly large data instances, i.e., with up to 150 wavelengths, in few minutes of computing times.**

We investigate how much bandwidth is wasted when no lightpath rearrangement is allowed, and compare it with the number of lightpath rerouting it requires in order to fully maximize the grade of service (GoS). Experiments are conducted on several data instances with up to 150 wavelengths. Results show that the amount of lightpath rearrangement varies with the size of the network, but in any case, remains very small in comparison to the amount of wasted bandwidth if not done.

I. Introduction

Wide-area WDM (Wavelength Division Multiplexing) networks built on the concept of Routing and Wavelength Assignment (RWA) are envisioned to form the backbone component of the all-optical network infrastructure. RWA utilizes optical connections called lightpaths, which traverse multiple fiber links and optical nodes, all with the same wavelength: this is the so-called wavelength continuity constraint. For each connection request, a lightpath is requested, provisioned, and when it is no longer required, torn down.

In dynamic WDM networks, connection arrivals and departures are stochastic in nature and connection provisioning is accomplished via online (or dynamic) RWA algorithms. Hence, the lightpath selection becomes sub-optimal. To enhance the Grade of Service (GoS), it may be desirable to seek a new RWA solution for all (or a subset) of the active connections. Migrating traffic from their current existing connection configuration to a new one is referred to as traffic migration or traffic configuration. Other terms, e.g., rerouting or lightpath rearrangement are also used in the literature for the action of altering the physical path and/or wavelength of an established connection. In this study, we will look at lightpath rearrangement when a new batch of connection requests comes, and minimize the number of lightpath rearrangement so as to keep the network in a state that corresponds to the maximization of the grade of service (GoS). Lightpath rearrangement is now made feasible with the use of colorless, directionless, and contentionless (CDC) reconfigurable optical add/drop multiplexers (ROADMs) in optical networks.

While reviewing the previous work, we will limit ourselves to the studies with no wavelength conversion, as it is now known that wavelength conversion is not of great help for maximizing the GoS ([1]–[3]), in spite of studies still advertising some benefit of wavelength conversion, e.g., [4]. Indeed, authors looking at exact methods usually conclude that wavelength conversion does not help, while authors considering heuristics have an opposite conclusion. Many studies have been conducted on rerouting or lightpath rearrangement, all of them with heuristics. The early ones were limited to rings or torus (see, e.g., [5]). Other studies look at general mesh networks (see, e.g., [6], [7]) with many of them made before 2000, with the consideration of shortest paths only, and sometimes with a unique arbitrarily chosen shortest path for a given node pair, while, as observed in [8], several shortest paths do exist in most networks, and even more second shortest paths that are only one hop longer than the shortest paths. More recently, some authors look at the cases with scheduling [9], [10], impairment or grooming [11] considerations.

II. Statement of the Dynamic RWA Problem

A. Generalities

Consider a WDM optical network represented by a multigraph $G = (V, L)$ with node set V indexed by v, where each node is associated with a node of the physical network, and with link set L indexed by ℓ where each link is associated with a fiber link of the physical network: the number of links from v to v' is equal to the number of fibers supporting traffic from v to v'. Connections and fiber links are assumed to be directional, and the traffic to be asymmetrical. The set of wavelengths is denoted by Λ, and indexed by λ with $W = |\Lambda|$.

The RWA problem can be considered under two different traffic assumptions. Firstly, the static RWA problem that applies to the case in which the set of connections is known in advance, and a lightpath must be established for each connection, with a lightpath being defined by the combination of a routing path p and a wavelength λ, so that no two paths sharing a link of G are assigned the same wavelength. Secondly, the dynamic RWA problem, in which connection requests arrive dynamically and remain for some amount of time before departing. Very often, optical connections are leased for long periods of time (e.g., weeks or months), and thus new connection requests come with significant lead time to set-up. In addition, they are often configured manually. In this study, we limit ourselves to incremental traffic and will assume new connection requests come in batches. Our objective is to investigate further the spectrum usage under different traffic increment rate and lightpath rearrangement assumptions.

In the context of on-line or dynamic traffic, we look at the traffic increase over a set T of time periods, so that, at each time period $t \in T$, we have: *(i)* the overall set of traffic demand at time t: $D^t = (D^t_{sd})_{(v_s,v_d) \in \mathcal{SD}^t}$ where $\mathcal{SD}^t = \{(v_s, v_d) : D^t_{sd} > 0\}$, *(ii)* the set of new requests at time t that are described by a $n \times n$ matrix D^{NEW} where $D^{\text{NEW}}_{sd} = D^t_{sd} \backslash \text{GoS}^{t-1}_{sd}$ defines the number of newly requested connections from v_s to v_d at time t, and GoS^{t-1}_{sd} the set of granted requests from v_s to v_d out of D^{t-1}_{sd}. *(iii)* the set of legacy requests, which are already provisioned, and described by a $n \times n$ matrix $D^{\text{LEG}} = (\text{GoS}^{t-1}_{sd})_{(v_s,v_d) \in \mathcal{SD}^{t-1}}$.

Let $\Lambda^{\text{USED}}_{t-1}$ be the set of used wavelengths for provisioning the traffic up to time period $t-1$, i.e., GoS^{t-1}. All wavelengths are assumed to have the same transport capacity. Let $\omega^+(v)$ (resp. $\omega^-(v)$) be the set of outgoing (resp. incoming) fiber links at node v.

B. Incremental Traffic without Lightpath Rearrangement

The dynamic RWA problem without rerouting can then be formally stated as follows: given a multigraph G corresponding to a WDM optical network, and a set of requested connections, find a suitable lightpath (p, λ) for each new incoming connection, with no lightpath rearrangement of an already provisioned request. We study the objective of minimizing the blocking rate, that is equivalent to maximizing the number of granted connections (also called Grade of Service or GoS for short), leading to the so-called dynamic max-RWA problem.

The provisioning of the new connection requests (D^{NEW}) need to be made using lightpaths that do not conflict with those already used for the connection requests of D^{LEG}. The new lighpaths can be defined using either the wavelengths already activated for the provisioning of D^{LEG}, or additional wavelengths made available for the new incoming traffic.

C. Incremental Traffic with Lightpath Rearrangement

Under the scenario of lightpath rearrangement, the objective is to provision a new batch of connection requests while allowing some minimum lightpath rearrangement in order to maximize the GoS. Again, new requests can be provisioned either on the already used wavelengths ($\Lambda^{\text{USED}}_{t-1}$) if enough spare resource is available, or on an additional wavelength as long as we do not exceed the number of available wavelengths (W). Lightpath rearrangements consist in either modifying the wavelength of an existing lightpath, or considering a new routing and wavelength.

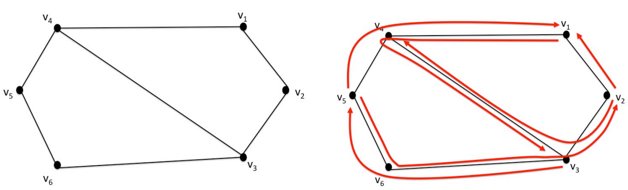
III. Optimization Models

We now describe the optimization models for dynamic RWA, starting with the notations common to the two models (Section III-A). Next we present the model without lightpath rearrangement (Section III-B), and then with minimum lightpath rearrangement (Section III-C).

A. Notations and Definitions

The optimization models rely on the concept of configurations, where a configuration is defined by a set of lightpaths, all associated with the same wavelength, see Figure 1. Consequently, in each wavelength configuration, routes must be pairwise link disjoint. A configuration c is formally represented by a non negative vector a^c such that $a^c_{sd} =$ number of connection requests from v_s to v_d that are supported by configuration c.

We use two sets of variables. The first set of variables, z_c, enables the selection of the best configurations and of their number of occurrences (i.e., to how many wavelengths they apply). The second set of variables, y_{sd}, determine the number of granted requests for node pair $(v_s, v_d) \in \mathcal{SD}^{\text{NEW}}$ (no rearrangement) or $(v_s, v_d) \in \mathcal{SD}^t$ (with rearrangement). We then have $0 \leq y_{sd} \leq D^t_{sd} - \text{GoS}^{t-1}_{sd}$ for $(v_s, v_d) \in \mathcal{SD}^t$ if rearrangement is not allowed, and $\text{GoS}^{t-1}_{sd} \leq y_{sd} \leq D^t_{sd}$ for $(v_s, v_d) \in \mathcal{SD}^t$ otherwise.



(a) Physical Topology (b) A Potential Wavelength Configuration

Fig. 1. Physical Network and a Potential Wavelength Configuration

B. Dynamic RWA with No Lightpath Rearrangement

In time period t of dynamic RWA problem, traffic requests of D^{t-1} are already provisioned with the wavelengths of $\lambda \in \Lambda^{\text{USED}}_{t-1}$. For $\lambda \in \Lambda^{\text{USED}}_{t-1}$, we define a set of configurations C_λ such that each configuration of $c \in C_\lambda$ contains the legacy lightpaths associated with D^{t-1} in addition to some new lightpaths associated with $D^{\text{NEW}} = D^t \setminus \text{GoS}^{t-1}$. Note that z_c associated with those wavelength configurations is a decision variable as each set C_λ is associated with a specific wavelength. As we might have additional available wavelengths, let C be the set of configurations associated with a generic wavelength $\lambda \in \Lambda \setminus \Lambda^{\text{USED}}_{t-1}$.

The optimization model ($D^{t-1} \rightarrow D^t$) can be written:

$$\max \sum_{(v_s,v_d) \in \mathcal{SD}^{\text{NEW}}} y_{sd} \tag{1}$$

subject to:

$$\sum_{c \in C_\lambda} z_c \leq 1 \qquad \lambda \in \Lambda^{\text{USED}}_{t-1} \tag{2}$$

$$\sum_{c \in C} z_c \leq W - |\Lambda^{\text{USED}}_{t-1}| \tag{3}$$

$$y_{sd} \leq \sum_{c \in C \cup \bigcup\limits_{\lambda \in \Lambda^{\text{USED}}_{t-1}} C_\lambda} a^c_{sd} z_c \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}} \tag{4}$$

$$y_{sd} \leq D^t_{sd} - \text{GoS}^{t-1}_{sd} \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}} \qquad (5)$$

$$z_c \in \{0,1\} \qquad c \in C_\lambda, \lambda \in \Lambda^{\text{USED}}_{t-1} \qquad (6)$$

$$z_c \in \mathbb{Z}^+ \qquad c \in C \qquad (7)$$

$$y_{sd} \geq 0 \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}}. \qquad (8)$$

For $\lambda \in \Lambda^{\text{USED}}_{t-1}$, constraints (2) restrict the number of selected configurations to be at most 1. Constraints (3) determine the number of new required wavelength configurations (or wavelengths) and make sure it does not exceed the number of remaining available wavelengths. Constraints (4) and (5) determine the number of granted connections without allowing to exceed the demand. Last three sets of constraints determine the domain of the variables.

C. Dynamic RWA with Lightpath Rearrangement

In order to improve the grade of service in a dynamic RWA problem, one might allow some lightpath rearrangement taking into account that new connection requests come with enough lead time to set-up. In such a case, the objective is to minimize the number of such rearrangements, while achieving a grade of service as close as possible to the one of the static max-RWA problem. Therefore, at a time period t, not only we provision new traffic requests, but we also allow some lightath rearrangement if it helps increasing the GoS. For $\lambda \in \Lambda^{\text{USED}}_{t-1}$, let γ_c be the total number of rearranged lightpaths in configuration $c \in C_\lambda$. The penalty for every unit of rearrangement is indicated by PENAL. The resulting optimization model can be written as follows.

$$\max \sum_{(v_s,v_d) \in \mathcal{SD}^t} y_{sd} - \text{PENAL} \sum_{\lambda \in \Lambda^{\text{USED}}_{t-1}} \sum_{c \in C_\lambda} \gamma_c z_c \qquad (9)$$

$$\text{subject to:} \quad (2) - (4), (6) - (8) \qquad (10)$$

$$\text{GoS}^t_{sd} \leq y_{sd} \leq D^t_{sd} \qquad (v_s, v_d) \in \mathcal{SD}^t. \qquad (11)$$

Constraints (11) bound the number of granted requests and ensure that all previously granted traffic requests are still provisioned, subject to some possible lightpath rearrangement.

IV. Solution of the Optimization Models

The models proposed in Section III have an exponential number of variables, and therefore are not scalable if solved using classical ILP (Integer Linear Programming) tools. We therefore recourse to decomposition techniques (column generation) in order to consider *implicitly* defined constraint matrices, leading then to scalable solution schemes.

A. Combination of Column Generation and ILP Tools

We use the column generation techniques (see Chvatal [12] for more details on those techniques) in order to solve the linear relaxation of the models of Section III and then deduce an ε-optimal ILP solution using the optimal linear programming solution.

Column generation method allows the exact solution of the linear relaxation of models (1) - (8) and (9) - (11), i.e., where constraints $z_c \in \mathbb{Z}^+$ are replaced by $z_c \geq 0$, for $c \in C$, and $z_c \in \{0,1\}$ are replaced by $0 \leq z_c \leq 1$, for $c \in C_\lambda$ for $\lambda \in \Lambda^{\text{USED}}_{t-1}$. It consists in solving alternatively a restricted master problem (the models of III with a very limited number of columns/variables) and the pricing problem (generation of a new wavelength configuration) until the optimality condition is satisfied (i.e., no wavelength configuration with a negative reduced cost, see again [12] if not familiar with linear programming concepts and the definition of a reduced cost). In other words, when a new wavelength configuration is generated, it is an improving one if, when added to the current restricted master problem, its addition implies an improvement of the optimal value of the current restricted master problem. This condition, indeed an optimality condition, can be easily checked with the sign of the reduced cost, denoted by $\overline{\text{COST}}$, see (16) for its expression, of variables z_c.

Once the optimal solution of the LP (Linear Programming) relaxation ($z^\star_{\text{LP}}$) has been reached, we solve exactly the last restricted master problem, i.e., the restricted master problem of the last iteration in the column generation solution process, using a branch-and-bound method, leading then to an ε-optimal ILP solution ($\tilde{z}_{\text{ILP}}$), where $\varepsilon = \frac{z^\star_{\text{LP}} - \tilde{z}_{\text{ILP}}}{z^\star_{\text{LP}}}$. Branch-and-price methods can be used to find optimal solutions, if the accuracy (ε) is not satisfactory, see, e.g., [13], [14].

B. Model with no Lightpath Rearrangement

1) Overview of the Solution Scheme: Let $\lambda\mathcal{P}$ be the set of lightpaths used to grant connections for the legacy traffic. At time period t, the set of available links L^t_λ for wavelength λ, is: $L^t_\lambda = L \setminus \{\ell \in L : \exists p \text{ with } \ell \in p \text{ and } (p, \lambda) \in \lambda\mathcal{P}\}$.

The proposed algorithm for solving dynamic max-RWA problem generates new augmenting configurations, using a path and a link mathematical formulation that serve as configuration generators. The two resulting pricing problems, or equivalently, wavelength configuration generators, are called PP^{PATH} and PP^{LINK}, respectively. They are called in sequence as illustrated in Figure 2, always PP^{PATH} before PP^{LINK} as PP^{PATH} requires less computing times than PP^{LINK} to generate an improving wavelength configuration if one exists, and then always $\text{PP}^{\text{LINK}}_\lambda$ and $\text{PP}^{\text{PATH}}_\lambda$ for $\lambda \in \Lambda^{\text{USED}}_{t-1}$, before the generic PP^{LINK} and PP^{PATH} for the additional wavelengths, as long as they generate improving wavelength configurations.

Each $\text{PP}^{\text{PATH}}/\text{PP}^{\text{PATH}}_\lambda$ considers a set of restricted paths in order to generate a new improving configuration. While we could restrict the set of paths to the set of shortest paths, we found out in [8] that it is worth adding few additional k-shortest paths. Consequently, we use the conclusions of [8] in order to define the set of paths in $\text{PP}^{\text{PATH}}/\text{PP}^{\text{PATH}}{}_\lambda$.

After the insertion of a first set of initial configurations capturing the provisioning of the legacy traffic (i.e., of D^{t-1}), the column generation algorithm alternately solves the restricted master problem (i.e., models (1) - (8) and (9)-(11) with a very small number of variables/columns) and the pricing problems $\text{PP}^{\text{PATH}}/\text{PP}^{\text{PATH}}_\lambda$ and $\text{PP}^{\text{LINK}}/\text{PP}^{\text{LINK}}_\lambda$, until the optimality condition is satisfied. In choosing the wavelength for consideration in each iteration, a round robin approach is used. The strategy is as follows. Firstly, the algorithm iterates through already provisioned wavelengths, until no other improving configurations are found. Then, it proceeds to the new wavelengths and iteratively tries to find new improving configurations. When no such configuration can be found, the algorithm goes back to the wavelengths in the set $\Lambda_{t-1}^{\text{USED}}$ and repeats until the stopping condition is fulfilled, i.e., the reduced cost of all pricing problems is positive, see again the flowchart of Figure 2 for an overview of the column generation algorithm.

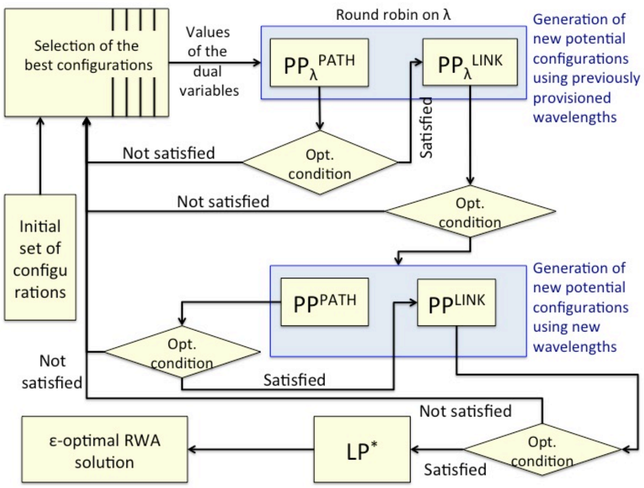


Fig. 2. Solution approach for dynamic requests

2) PP$^{\text{PATH}}$ and PP$_\lambda^{\text{PATH}}$: In the pricing problem written with a path formulation, for every wavelength $\lambda \in \Lambda_{t-1}^{\text{USED}}$ we provide a set $P_{sd}^{\lambda,t}$ of paths for each source and destination pair of nodes, which only contain links belonging to L_λ^t. Let SP_{sd}^k be the set of paths with the k smallest value. Then,

$$P_{sd}^{\lambda,t} = \bigcup_{i=1}^{k} \{p \in SP_{sd}^i : p \text{ only contains link(s) } \ell \in L_\lambda^t\},$$

for some selected k (see [8] for more details).

PP$_\lambda^{\text{PATH}}$ uses one set of decision variables: $\beta_p^{sd} = 1$ if path p is used in the wavelength configuration under construction, 0 otherwise. It can be written as follows:

$$\max \quad -u^{(2)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{p\in P_{sd}^{\lambda,t}} \beta_p^{sd} u_{sd}^{(4)} \tag{12}$$

subject to:

$$\sum_{(v,v_d)\in\mathcal{SD}^t} \sum_{p\in P_{sd}^{\lambda,t}} \delta_\ell^p \beta_p^{sd} \le 1 \qquad \ell \in L_\lambda^t \tag{13}$$

$$\sum_{p\in P_{sd}^{\lambda,t}} \beta_p^{sd} \le D_{sd}^t - \text{GoS}_{sd}^{t-1} \qquad (v_s,v_d) \in \mathcal{SD}^{\text{NEW}} \tag{14}$$

$$\beta_p^{sd} \in \{0,1\} \qquad p \in P_{sd}^{\lambda,t}, (v_s,v_d) \in \mathcal{SD}^{\text{NEW}}. \tag{15}$$

We guarantee paths that are pairwise link disjoint thanks to constraints (13), in which δ_ℓ^p is a binary value representing the presence of link ℓ in path p. Constraints (14) prevent from exceeding the traffic demand. Constraints (15) define the domain of variables β_p^{sd}.

Correspondence between variables of the pricing problem and coefficients of the master problem: $a_{sd} = \sum_{p\in P_{sd}^{\lambda,t}} \beta_p^{sd}$.

For $\lambda \in \Lambda \setminus \lambda_{t-1}^{\text{USED}}$, PP$^{\text{PATH}}$ is very similar to PP$_\lambda^{\text{PATH}}$: constraints (13) and (15) are written for all $\ell \in L$, and the reduced cost has to be updated: replace $u^{(2)}$ by $u^{(3)}$.

3) PP$^{\text{LINK}}$ and PP$_\lambda^{\text{LINK}}$: As always with the column generation method, the objective of the pricing problem is the reduced cost ($\overline{\text{COST}}_c^{\text{LINK}}$) of variable z_c. In order to alleviate the notations, index c will be omitted in the remainder of this section.

We first describe PP$_\lambda^{\text{LINK}}$ for $\lambda \in \Lambda_{t-1}^{\text{USED}}$. Let $u^{(2)}$ and $u_{sd}^{(4)}$ be the values of the dual variables associated with constraints (2) and (4) in the optimal solution of the linear relaxation of the current restricted master problem. Consider the following variables: $\alpha_\ell^{sd} = 1$ if link $\ell \in L_\lambda^t$ is used in a route from v_s to v_d, 0 otherwise.

For $\lambda \in \Lambda_{t-1}^{\text{USED}}$, PP$_\lambda^{\text{LINK}}$ can be written as follows:

$$\max \quad -u^{(2)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd} u_{sd}^{(4)} \tag{16}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}^t} \alpha_\ell^{sd} \le 1 \qquad \ell \in L_\lambda^t \tag{17}$$

$$\sum_{\ell\in\omega^+(v)} \alpha_\ell^{sd} = \sum_{\ell\in\omega^-(v)} \alpha_\ell^{sd} \qquad (v_s,v_d) \in \mathcal{SD}^{\text{NEW}}, \; v \in V \setminus \{v_s,v_d\} \tag{18}$$

$$\sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd} \le D_{sd}^t - \text{GoS}_{sd}^{t-1} \qquad (v_s,v_d) \in \mathcal{SD}^{\text{NEW}} \tag{19}$$

$$\sum_{\ell\in\omega^-(v_s)} \alpha_\ell^{sd} = \sum_{\ell\in\omega^+(v_d)} \alpha_\ell^{sd} = 0 \qquad (v_s,v_d) \in \mathcal{SD}^{\text{NEW}} \tag{20}$$

$$\alpha_\ell^{sd} \in \{0,1\} \qquad \ell \in L_\lambda^t, (v_s,v_d) \in \mathcal{SD}^{\text{NEW}}. \tag{21}$$

Constraints (17) prevent wavelength clashes, i.e., that a link cannot be traversed by more than one route in any given wavelength configuration. Routes are established with the help of the flow conservation constraints (18): if no route is selected for node pair (v_s,v_d), then $\alpha_\ell^{sd} = 0$ for all links $\ell \in L_\lambda^t$, otherwise, the sum of the outgoing flow values at the source node ($\sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd}$) gives the number of link-disjoint routes from v_s to v_d in the wavelength configuration under construction. Constraints (20) prevent loops around the source or the destination nodes from arising. Constraints (21) define the domain of variables α_ℓ^{sd}. Correspondence between variables of the pricing problem and coefficients of the master problem: $a_{sd} = \sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd}$.

For $\lambda \in \Lambda \setminus \lambda_{t-1}^{\text{USED}}$, PP$^{\text{LINK}}$ is very similar to PP$_\lambda^{\text{LINK}}$: constraints (17) and (21) are written for all $\ell \in L$, and the reduced cost has to be updated in order to consider $u^{(3)}$ rather than $u^{(2)}$.

C. Model with Lightpath Rearrangement

Assuming rearrangement of lightpaths is allowed, the sets of available links and paths for all wavelengths remain identical, i.e., for all $\lambda \in \lambda$ and time periods t, $L^t_\lambda = L$ and $P^{\lambda,t}_{sd} = P_{sd}$ for all node pairs.

The following modifications to $\text{PP}^{\text{LINK}}_\lambda$ provides a configuration generator for the current problem. Denote by $\bar{P}^{\lambda,t-1}_{sd}$, $(v_s, v_d) \in \mathcal{SD}^{t-1}$, the set of assigned paths for legacy pair (v_s, v_d) using wavelength $\lambda \in \Lambda^{\text{USED}}_{t-1}$. For every $p \in \bar{P}^{\lambda,t-1}_{sd}$, γ^p is a binary variable equal to 1 if path $p \in \bar{P}^{\lambda,t-1}_{sd}$ is modified in the new configuration for λ. The reduced cost is as follows:

$$\max \quad - \text{PENAL} \sum_{(v_s,v_d)\in\mathcal{SD}^{t-1}} \sum_{p\in\bar{P}^{\lambda,t-1}_{sd}} \gamma^p - u^{(2)} + \sum_{(v_s,v_d)\in\mathcal{SD}^t} \sum_{\ell\in\omega^+(v_s)} \alpha^{sd}_\ell u^{(4)}_{sd} \qquad (22)$$

In order to determine the number of modified lightpaths in $\text{PP}^{\text{LINK}}_\lambda$, the next set of constraints are added:

$$\alpha^{sd}_\ell \geq 1 - \gamma^p \quad p \in \bar{P}^{\lambda,t-1}_{sd}, (v_s, v_d) \in \mathcal{SD}^{t-1} \qquad (23)$$

$$\gamma^p \in \{0, 1\} \qquad p \in \bar{P}^{\lambda,t-1}_{sd}, (v_s, v_d) \in \mathcal{SD}^{t-1}. \qquad (24)$$

Considering the uniqueness of paths in every configuration, Constraints (23) determine the values of γ^p, $p \in \bar{P}^{\lambda,t-1}_{sd}$, by checking whether all its consisting links contribute to the provisioning of pair (v_s, v_d). Correspondence between variables of the pricing problem and the coefficients γ in the master problem becomes: $\gamma = \sum_{p\in P^{\lambda,t-1}} \gamma^p$.

Similarly, $\text{PP}^{\text{PATH}}_\lambda$ is modified as follows. Let γ_{sd}, be the number of modified lightpaths in configuration $c \in C_\lambda$ for wavelength $\lambda \in \Lambda^{\text{USED}}_{t-1}$. The reduced cost of $\text{PP}^{\text{PATH}}_\lambda$ is then:

$$\max \quad - \text{PENAL} \sum_{(v_s,v_d)\in\mathcal{SD}^{t-1}} \gamma_{sd} - u^{(2)} + \sum_{(v_s,v_d)\in\mathcal{SD}^t} \sum_{\ell\in\omega^+(v_s)} \alpha^{sd}_\ell u^{(4)}_{sd} \qquad (25)$$

The number of rearranged paths is computed using following constraints:

$$|\bar{P}^{\lambda,t-1}_{sd}| - \sum_{\substack{p\in \\ \bar{P}^{\lambda,t-1}_{sd}\cap P_{sd}}} \beta^{sd}_p \leq \gamma_{sd} \quad (v_s, v_d) \in \mathcal{SD}^{t-1} \qquad (26)$$

$$\gamma_{sd} \in \mathbb{Z}^+ \qquad (v_s, v_d) \in \mathcal{SD}^{t-1}. \qquad (27)$$

Correspondence between variables of the pricing problem and the coefficients in the master problem is $\gamma = \sum_{(v_s,v_d)\in\mathcal{SD}^{t-1}} \gamma_{sd}$.

V. Numerical Results

A. Data Instances

We run experiments on five different networks: NSFNET [15], USANET [16], GERMANY [15], NTT [17], and ATT [18], and whose characteristics are described in Table I. All computational results have been obtained with running the programs on a server with the help of CPLEX [19] (Version V12.6.2) for solving the (integer) linear programs. Programs never used more than 2Gb memory and 2 CPUs. Traffic instances are described in [8], and correspond to a mixed of randomly generated and of realistic traffic requests.

In Table I, for each data instance, we provide the number of nodes, links, available wavelengths, node pairs with requests ($|\mathcal{SD}|$) and the overall number of traffic requests ($D = \sum_{\{v_s,v_d\}\in\mathcal{SD}} d_{sd}$), together with the traffic distribution.

TABLE I
Characteristics of the Datasets

Data instances	$\lvert V\rvert$	$\lvert L\rvert$	W	$\lvert\mathcal{SD}\rvert$	$\sum_{(v_s,v_d)\in\mathcal{SD}} D_{sd}$	Traffic distribution μ	Traffic distribution σ
NSF_{30}	14	40	30	141	436	3.1	1.4
NSF_{75}			75	182	1,371	7.5	2.4
NSF_{115}			115	182	2,194	12.1	2.7
USA_{75}	24	88	75	455	1,336	2.9	2.4
USA_{125}			125	541	2,422	4.5	1.9
USA_{150}			150	552	3,509	6.4	2.2
GER_{100}	50	176	100	660	2,365	3.6	6.2
GER_{130}			130		3,041	6.2	4.6
GER_{150}			150		4,989	8.6	6.3
NTT_{42}	55	144	42	338	1,038	3.1	1.4
NTT_{50}			50	452	1,362	3.0	1.4
NTT_{150}			150	452	5,684	12.6	7.4
ATT1_{20}	90	274	20	272	359	1.3	0.7
ATT2_{113}	71	350	113	2,869	2,918	1.0	0.7

In order to simulate the incremental traffic, the connection requests of every data set are divided into smaller sets, such that $\mathcal{SD}^{t-1} \subseteq \mathcal{SD}^t$ and for every pair $(v_s, v_d) \in \mathcal{SD}^t$, $D^{t-1}_{sd} \leq D^t_{sd}$. This is done in consecutive steps of size δ: in each time period, δ unit traffic requests are selected randomly, from the original demand set $\mathcal{SD}$ at time $t = 0$.

B. Wasted Spectrum Utilization

We use four different values of δ, 10, 50, 200 and 500, and report the results in Table II for the first dynamic RWA model, with no lightpath rearrangement. The results for the static RWA are obtained by considering all the traffic requests at once at time period $t = 1$. For dynamic RWA, after each addition of δ unit requests, we optimize their provisioning while preserving the provisioning of the legacy requests. We keep adding batches of δ requests until all requests (see Table I) have been added.

We observe that, when there is no lightpath rearrangement, GoS with dynamic traffic is much less than the optimum GoS of the static case. The difference varies with the step size δ. In every data set, up to a turning point, a greater step size affects more the set of available lightpaths and causes the GoS to drop. From that turning point on, sets of new traffic requests become big enough and GoS behavior is closer to the one of the static traffic. For instance, in GER_{130}, GoS decreases as the step size grows from 50 to 200. After this point, with increasing the step size, GoS starts to improve.

TABLE II
GRADE OF SERVICE - NO LIGHTPATH REARRANGEMENT.

Data instances	GoS				
	Static	Dynamic			
		$\delta = 10$	$\delta = 50$	$\delta = 200$	$\delta = 500$
NSF_{30}	96.6	83.7	71.6	74.1	96.6
NSF_{75}	89.6	71.3	62.3	56.6	63.0
NSF_{115}	87.0	70.6	61.3	52.8	59.4
USA_{75}	92.9	87.0	78.6	71.2	78.5
USA_{125}	90.9	80.6	76.5	66.3	68.7
USA_{150}	84.8	73.7	65.8	57.6	59.3
GER_{100}	94.9	82.7	75.7	68.9	76.3
GER_{130}	95.0	81.8	71.6	67.9	71.7
GER_{150}	79.8	60.7	55.0	44.7	48.5
NTT_{42}	100.0	98.1	96.3	93.3	96.6
NTT_{50}	100.0	90.1	89.4	85.2	89.7
NTT_{150}	96.8	76.4	73.9	59.9	63.5
ATT_{20}	98.6	81.3	75.5	86.1	98.6
ATT_{113}	99.5	84.5	80.0	85.5	95.5

C. Minimizing the Lightpath Rearrangements

We summarize in Table III the results obtained when considering some lightpath rearrangement. Therein, PENAL $= 0.1$. For each problem instance, we report the maximum GoS (absolute and percentage values), the cumulative number of rearranged lightpaths (absolute and percentage values), and then the average number of lightpath rearrangements per time period. As can be observed, this last number is quite small, i.e., always smaller than 8%. This shows that, by allowing the rearrangement of lightpaths, one can achieve a GoS very close to that of the static traffic, while paying the cost of rearranging a very small percentage of already established lightpaths.

TABLE III
DYNAMIC RWA WITH MINIMUM REARRANGEMENT

Data instances	δ	GoS		Cumulative disruption		Average disruption
		#	%	#	%	%
NSF_{30}	50	415	95.2	78	18.8	3.7
NSF_{75}	200	1,222	89.0	215	17.6	1.9
NSF_{115}	500	1,920	87.5	361	18.8	4.3
USA_{75}	50	1,234	92.4	351	28.4	1.1
USA_{75}	200	1,234	92.4	314	25.4	4.1
USA_{125}	100	2,160	89.2	573	26.5	1.4
USA_{125}	500	2,158	89.1	447	20.7	5.6
USA_{150}	200	2,855	81.4	802	28.1	1.9
USA_{150}	500	2,841	81.0	695	24.5	3.1
GER_{100}	250	2,206	93.3	505	22.9	2.9
GER_{100}	500	2,201	93.1	392	17.8	4.7
GER_{130}	250	2,861	94.1	856	29.9	2.8
GER_{130}	500	2,848	93.7	670	23.5	4.4
GER_{150}	250	4,507	79.5	1,712	38.0	1.9
GER_{150}	500	4,483	79.1	1,303	29.1	3.0
NTT_{42}	200	1,038	100.0	9	0.9	0.2
NTT_{50}	200	1,362	100.0	24	1.8	0.8
NTT_{150}	500	5,431	95.5	111	2.0	0.2
$ATT1_{20}$	50	359	100.0	160	44.6	7.3
$ATT2_{113}$	250	2,898	99.3	840	29.0	3.3

VI. CONCLUSIONS

We found out that the level of rearrangement that is required for maintaining a network in the state of maximum GoS under dynamic traffic, is indeed very small, assuming incremental traffic. Future work will explore whether lightpath rearrangement implies additional regenerators, if yes, how many. We will also investigate dynamic traffic not only with incremental traffic, but also with some torn down traffic requests.

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