

# Scalable Elastic Optical Path Networking Models

(Invited Paper)

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**Abstract**—In order to face the steady growth of the optical networks, network operators are now moving to elastic optical networking, with the challenge of optimizing the spectrum usage through the so-called routing and spectrum assignment (RSA) problem. It is a much more difficult problem than the classical routing and wavelength assignment (RWA) problem, for which practical algorithms are now available for solving realistic data instances.

Several authors have already investigated the routing and spectrum assignment problem, with either heuristics or exact algorithms. However, there is still a gap between the size of the instances that can be solved using those algorithms, and the sizes of the industry RSA instances. The objective of this study is to reduce the gap between the two, using a new mathematical modelling, and to compare its performance with the best previous algorithms/models on various realistic data instances.

## I. INTRODUCTION

Network operators are very much concerned by the efficient utilization of the already deployed network capacity. Current wavelength-routed optical path networks require the full allocation of wavelength capacity to lightpaths between node pairs even when the traffic is not sufficient to fill the entire capacity of wavelength. It leads to inefficient capacity utilization, an issue expected to become even more significant with the deployment of higher capacity WDM networks, and driven by the imminent optical capacity crunch [1].

In order to address this drawback, thanks to the Orthogonal Frequency Division Multiplexing (OFDM) modulation technique [2], [1], flexible optical networks allow the allocation of bandwidth resources with granularity finer than a wavelength, leading to sub-wavelength but also to super-wavelength lightpaths, see Figure 1.

We consider the planning problem of an OFDM optical network, where connections are provisioned for their requested rate by elastically allocating spectrum using a variable number of OFDM subcarriers and choosing an appropriate modulation level taking into account the transmission distance. This corresponds to the so-called static RSA problem. A number of heuristic algorithms have been proposed, e.g., [3]. However, while heuristics allow faster solution of difficult combinatorial problems, they provide solutions with no estimation of their quality, i.e., how far they are from an optimal solution.

For exact solution of the RSA problem, several authors investigated ILP formulations with and without explicit modulation concerns. Early proposed formulations are not scalable

and work only on toy examples with a very limited number of slots, see, e.g., [4]. Even if this is not a major difficulty, few studies consider the guard band requirement in their mathematical models, except for, e.g., [5]. Then, some more recent studies consider some column generation models with different decomposition schemes. Ruiz *et al.* [6] consider a heuristic decomposition scheme with the implicit enumeration of lightpaths, and are able to solve data instances with up to 96 slots, and an overall demand distributed over a set of up to 180 node pairs in the Spain network (21 nodes, 35 links). More recently, Moataz [7] investigated a decomposition based on slot configurations, which gather all the lightpaths using a given slot, in an attempt to generalize the wavelength configurations used in the context of the RWA problem [8], [9]. However, results were quite disappointing.

The paper is organized as follows. In the next section, we provide a detailed statement of the RSA problem, and of the notations we will use in the sequel. In Section III, we describe a new formulation, based on the definition of groups of lightpaths with the same lowest slot. Solution scheme is explained in Section IV. Numerical results are presented in Section V. Conclusions and future work are discussed in the last section.

## II. STATEMENT OF THE RSA PROBLEM

Let us consider an elastic (flexgrid) optical network, represented by an undirected graph  $G = (V, L)$  with optical node set  $V$  (indexed by  $v$ ) and fiber link (edge) set  $L$  (indexed by  $\ell$ ). Connections and fiber links are assumed to be undirected, and the traffic to be symmetrical. The bandwidth is slotted into a set  $S$  (generic index  $s$ ) of spectrum slots (or slices). A guard band  $g$  (number of slots) is required between two spectrum contiguous allocations. The available bandwidth over every fiber link is defined by the overall number of spectrum slots (12.5 GHz steps in this study). The traffic is defined by a set  $K$  of requests where each request  $k \in K$  has a source ( $s_k$ ), a destination ( $d_k$ ) and a spectrum demand  $D_k$ , expressed in terms of a number of slots, requested to be contiguous. We assume that no regenerator is used, and therefore, the number of slots corresponds to the modulation that is compatible with the distance between  $s_k$  and  $d_k$  and that uses the minimum number of slots.

The RSA problem can be formally stated as follows: given a graph  $G$  corresponding to an elastic optical network, and a set

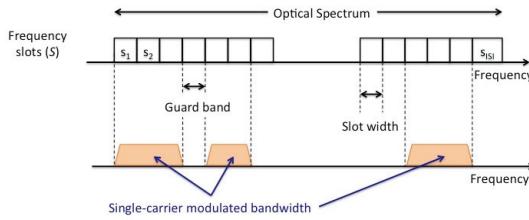


Fig. 1. Connection requests use a group of spectrum contiguous slots

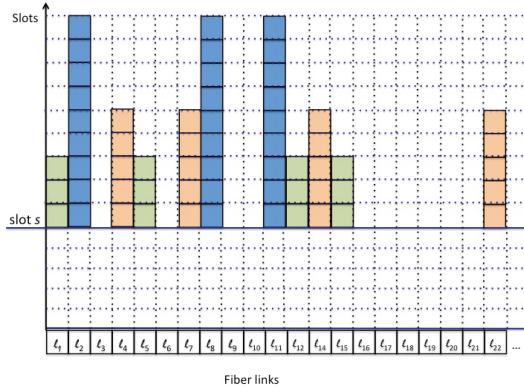


Fig. 2. Configuration example

of requested connections, find a path and a spectrum allocation for every request respecting the continuity and contiguity constraints. The objective is to minimize the blocking rate, that is equivalent to maximizing the number of accepted connections, leading to the max-RSA problem.

### III. CONFIGURATION OPTIMIZATION MODEL

We propose a new decomposition scheme that relies on lightpath configurations such that, for each configuration, blocks of consecutive slides are all starting at the same slot, see Figure 2 for an illustration of three lightpaths in the Spain network represented in Figure 4. Consequently, each configuration is indexed by  $s$ , the starting slot, and contains a set of lightpaths using one or more slots, but such that the first slot of each lightpath is  $s$  indexed. Whenever a request is granted in a configuration, it is for its overall bandwidth/slot requirement, with taking care of the slot contiguity constraint.

A configuration  $c \in C = \bigcup_{s \in S} C_s$  is characterized by:  $a_k^c$  that is equal to 1 if request  $k$  is provisioned in  $c$ , 0 otherwise, and  $a_{k,\ell}^{s,c}$  that is equal to one if  $k$  is provisioned with a lightpath going through link  $\ell$  and slot  $s$ , 0 otherwise. The model is written as follows:

$$\max \sum_{c \in C} \left( \sum_{k \in K} D_k a_k^c \right) z_c \quad (1)$$

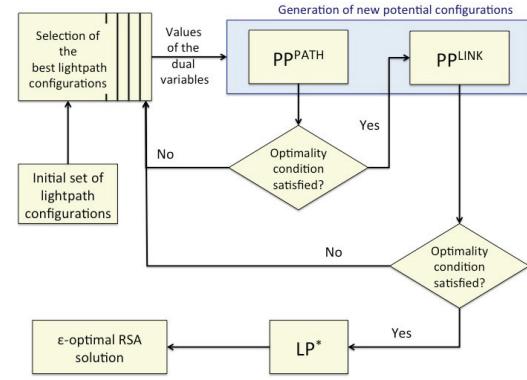


Fig. 3. Flowchart

$$\text{subject to: } \sum_{c \in C_s} z_c \leq 1 \quad s \in S \quad (2)$$

$$\sum_{c \in C} a_k^c z_c \leq 1 \quad k \in K \quad (3)$$

$$\sum_{c \in C} \sum_{k \in K} a_{k,\ell}^{s,c} z_c \leq 1 \quad s \in S, \ell \in L \quad (4)$$

$$z_c \in \{0, 1\} \quad c \in C \quad (5)$$

Constraints (2) allow at most one configuration per "starting" slot for the consecutive slot blocks. Constraints (3) ensure that each request is accepted at most once. Constraints (4) do not allow more than one lightpath going through a given link ( $\ell$ ) for a given slot ( $s$ ). Constraints (5) determine the domain of variables  $z_c$ .

### IV. SOLUTION SCHEME

As the number of variables of model (1) - (5) is exponential, we need to recourse to column generation techniques for solving the linear relaxation as in [6], and consequently to define the corresponding configuration generator problem (called pricing problem in the column generation literature). The flowchart of the solution scheme is depicted in Figure 3. In order to ease the solution of the configuration generator, we consider two formulations, a link one that can check thoroughly for new improving configurations, but that is computationally expensive, and a path one, that is more scalable, with a set of pre-computed paths, but which do not guarantee to find an improving configuration when one exists. The combination of the two pricing problems is described in the flowchart of Figure 3 in order to guarantee an optimal solution of the linear relaxation of (1) - (5).

In order to alleviate the presentation, pricing problems are written without taking care of the guard band. That last feature is easy to add: increase the slot requirement by one unit on the right endpoint, except when the last required slot reaches the right boundary of the spectrum.

#### A. $PP_{LINK}$ : Pricing Problem - Link Formulation

As always with the column generation method, the objective of the pricing problem (i.e., generator of new configurations) is

the reduced cost ( $\overline{\text{COST}}_c^{\text{LINK}}$ ) of variable  $z_c$ . In order to alleviate the notations, index  $c$  will be omitted in the remainder of this section.

Let  $u_s^{(2)}$ ,  $u_k^{(3)}$  and  $u_{s\ell}^{(4)}$  be the values of the dual variables associated with constraints (2), (3) and (4) in the optimal solution of the linear relaxation of the current restricted master problem (see the flowchart in Figure 3).

Let  $K_s$  be the set of requests that have the potential to be provisioned in  $\text{PP}_{\text{LINK}s}$ :

$$K_s = \{k \in K : s + D_k - 1 \leq |S|\}.$$

Consider the following set of variables:

$a_\ell^{k,s} = 1$  if link  $\ell$  is used in a route of width  $D_k$  slots from  $s_k$  to  $d_k$  such that the index of the lower slot is  $s$ , 0 otherwise.  
 $a_k = 1$  if request  $k$  is granted in the configuration under construction, 0 otherwise.

The link formulation of the Lightpath Configuration Generator, called  $\text{PP}_{\text{LINK}s}$ , can be written as follows:

$$\begin{aligned} \max \quad & \overline{\text{COST}}^{\text{LINK}} = \sum_{k \in K_s} a_k - u_s^{(2)} - \sum_{k \in K_s} u_k^{(3)} a_k \\ & + \sum_{k \in K_s} \sum_{s'=s}^{s+D_k-1} \sum_{\ell \in L} u_{s'\ell}^{(4)} a_\ell^{k,s'} \end{aligned} \quad (6)$$

subject to:

$$\sum_{k \in K_s} a_\ell^{k,s} \leq 1 \quad \ell \in L \quad (7)$$

$$\sum_{\ell \in \omega(s_k)} a_\ell^{k,s} = \sum_{\ell \in \omega(d_k)} a_\ell^{k,s} = a_k \quad k \in K_s \quad (8)$$

$$\sum_{\ell \in \omega(v)} a_\ell^{r,s} \leq 2 a_k \quad k \in K_s, v \in V \setminus \{s_k, d_k\} \quad (9)$$

$$\sum_{\ell' \in \omega(v) \setminus \{\ell'\}} a_\ell^{k,s} \geq a_{\ell'}^{r,s} \quad \ell' \in \omega(v), v \in V \setminus \{s_k, d_k\}, \quad k \in K_s \quad (10)$$

$$\sum_{i=1}^{D_k-1} a_\ell^{k,s+i} = a_\ell^{r,s} (D_k - 1) \quad \ell \in L, k \in K_s \quad (11)$$

$$a_\ell^{k,s'} \in \{0, 1\} \quad s' \in \{s, s+1, \dots, s+D_k-1\} \quad k \in K_s, \ell \in L \quad (12)$$

$$a_k \in \{0, 1\} \quad k \in K_s. \quad (13)$$

Constraints (7) prevent wavelength clashes, i.e., a link cannot be traversed by more than one route in any given lightpath configuration. Routes are established with the help of constraints (9) and (10): if no route is selected for request  $k$ , then  $a_\ell^{k,s} = 0$  for all links  $\ell \in L$ , otherwise,  $a_k = 1$  and intermediate nodes of the single path from  $s_k$  to  $d_k$  are identified. Constraints (11) take care of the slot contiguity constraints, and reserve adjacent slots in order to fulfill the slot demand of granted requests. Constraints (12) define the domain of variables  $a_\ell^{r,s'}$ , and constraints (13) define the domain of variables  $a_k$ .

### B. $\text{PP}_{\text{PATH}}$ : Pricing Problem - Path Formulation

In the path formulation, we provide a pre-computed set of paths for each source and destination pair  $(v_s, v_d)$  of nodes, which will be used for provisioning requests between  $v_s$  and  $v_d$ . We denote by  $P_k$  the set of paths associated with requests  $k$ . Selection of paths (how many for each node pair) is made as in the algorithms we developed for the Routing and Wavelength Assignment (RWA) problem, see [9]. The path formulation for the wavelength configuration generator is denoted by  $\text{PP}_{\text{PATH}}$ . It uses the set of decision variables:  $\beta_p^{k,s} = 1$  if path  $p$  is used in the wavelength configuration under construction for provisioning request  $k$ , 0 otherwise.

The path formulation of the Lightpath Configuration Generator, called  $\text{PP}_{\text{PATH}}$ , can be written as follows:

$$\begin{aligned} \max \quad & \sum_{k \in K_s} a_k - u_s^{(2)} - \sum_{k \in K_s} u_k^{(3)} a_k \\ & - \sum_{k \in K_s} \sum_{p \in P_k} \sum_{s'=s}^{s+D_k-1} u_{s'\ell}^{(4)} \delta_p^\ell \beta_p^{k,s'} \end{aligned} \quad (14)$$

subject to:

$$\sum_{k \in K_s} \sum_{p \in P_k} \beta_p^{k,s} \delta_p^\ell \leq 1 \quad \ell \in L \quad (15)$$

$$\sum_{p \in P_k} \beta_p^{k,s} = a_k \quad k \in K_s \quad (16)$$

$$\sum_{i=1}^{D_k-1} \beta_p^{k,s+i} = \beta_p^{k,s} (D_k - 1) \quad p \in P_k, k \in K_s \quad (17)$$

$$\beta_p^{k,s'} \in \{0, 1\} \quad s' \in \{s, \dots, s+D_k-1\} \quad p \in P_k, k \in K_s \quad (18)$$

$$a_k \in \{0, 1\} \quad k \in K_s. \quad (19)$$

Pairwise link disjointness for paths is guaranteed thanks to constraints (15), in which  $\delta_p^\ell$  is a binary parameter indicating whether link  $\ell$  belongs or not to path  $p$ . Constraints (16) enforces to choose only one lightpath per granted request. Constraints (17) ensure the slot contiguity requirement for each request. Constraints (18) and (19) define the domain of the variables.

Correspondence between variables of the pricing problem and coefficients of the master problem:

$$a_{k,\ell}^s = \sum_{p \in P_k} \sum_{s'=s}^{s+D_k-1} \delta_p^\ell \beta_p^{k,s'}.$$

## V. NUMERICAL RESULTS

We implemented the algorithms proposed in the previous section and tested them on the Spain network (21 nodes, 35 edges) as in [6]. All computational results have been obtained with running the programs on a on a 1.9-2.5GHz Core i5 machine with 4GB RAM running Windows 10, with the help of CPLEX (Version V12.6.2) for solving the (integer) linear programs. We generated several randomly generated

Traffic demand			# precomputed $k$ -shortest paths	$\varepsilon$ -optimal solution							load per link (%)		# cols	CPU time (sec.)
Total load (in Tbps)	D	S		Weighted GoS (Tbps)			Unweighted GoS	average # lightpaths per config.	$\mu$	$\sigma$				
Randomly generated (node pair) requests with a random number of slots drawn in $\{2, 4, \dots, 16\}$ slots														
7.45	35	80	639	7.45	6.70	90%	10%	30	86%	6.2	40.0	15.9	331	134
9.76	45	110	791	9.76	8.80	90%	10%	38	84%	6.7	35.5	10.7	365	177
10.70	60	156	812	10.70	9.45	88%	12%	51	85%	9.8	22.3	6.3	375	261
15.50	64	170	847	15.50	12.95	84%	16%	52	81%	7.7	28.9	9.2	477	630
15.10	70	236	956	15.10	13.10	87%	13%	59	84%	6.9	21.8	7.4	519	1,342
16.85	80	256	971	16.85	14.45	86%	14%	68	85%	7.7	23.1	5.8	622	1,419
Randomly generated (node pair) requests with a random number of slots drawn in $\{1, 2, \dots, 8\}$ slots														
3.67	35	50	451	3.67	3.17	86%	14%	30	86%	5.8	27.9	12.3	208	50
4.75	45	60	498	4.75	4.15	87%	13%	41	91%	4.8	34.3	12.4	326	86
6.77	60	75	904	6.77	5.75	85%	15%	50	83%	4.5	35.1	12.1	654	147
7.45	64	85	717	7.45	6.00	81%	19%	51	80%	7.4	31.7	11.3	380	176
7.37	70	100	729	7.37	6.17	84%	16%	57	81%	6.1	24.9	12.6	305	263
9.67	80	120	1,014	9.67	8.15	84%	16%	68	85%	7.2	29.0	8.7	356	323
11.95	112	150	1,301	11.95	10.22	86%	14%	98	88%	7.6	29.1	7.1	480	417
20.52	180	330	2,202	20.52	16.85	82%	18%	148	82%	7.1	20.8	4.6	635	1,606

data instances. Each instance is characterized by: *i*) a random selection of  $K$  source/destination pairs and *(ii)* a traffic profile in which each demand has a number of slots randomly generated in  $\{1, 2, \dots, 8\}$  or  $\{2, 4, \dots, 16\}$ . For computing the associated bandwidth, we assumed a slot width of 12.5 GHz and a spectral efficiency of 25 Gbps per slot as in [6].

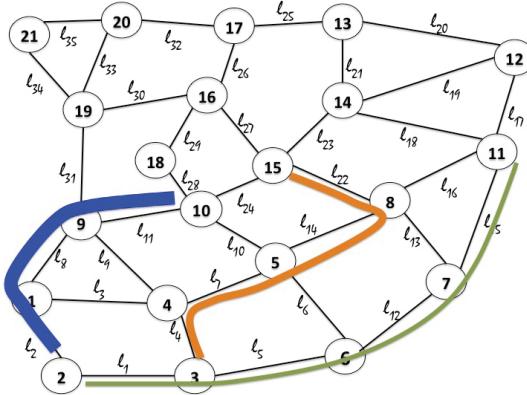


Fig. 4. Spain Network

While the largest instances that Ruiz *et al.* were able to solve had up to 180 demands and 96 available slots, we were able to solve instances with 180 requests and 330 available slots on the Spain network. The heuristic rule that we borrowed from [9] works well as it allowed us to reach the optimal LP relaxation value in much less time than [6], in addition to the fact that [6] used a more powerful computer than us (i.e., 2.4GHz Quad-core machine with 8GB RAM running Linux).

We observe that for all data sets, the optimal LP value is indeed equal to the input load, and can be reached fairly easily. Therefore, a challenge is to generate sufficiently good columns (i.e., lightpath configurations) in order to get good integer solutions, with a reasonable integrality gap ( $\varepsilon$ ). While  $\varepsilon$  is not very small, it is comparable or smaller than what has been obtained by [6].

## VI. CONCLUSION AND FUTURE WORK

We provided a new decomposition model for the RSA problem which outperformed the performances of the decomposition used in Ruiz *et al.* [6]. Improvements are still needed in order to solve realistic sized data instances. Future work will include improving the scalability of the proposed solution scheme, and the selection of the modulation. We might also consider the use of regenerators to enhance further the GoS at the expense of an additional cost.

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